

In 1882, using the same technique as Hermite, Ferdinand Lindemann proved that π was transcendental. While this was an incredibly important discovery, Kronecker was quite unimpressed and wrote to Lindemann,

Of what use is your beautiful investigation regarding π ? Why study such problems when even irrational numbers do not exist?

The transcendence of π is perhaps one of Lindemann's two greatest legacies in mathematics, the other being David Hilbert. It is no exaggeration to say that David Hilbert is one of the top five most important mathematicians of the 20th century. At the 2nd International Congress of Mathematicians in Paris, David Hilbert presented a talk entitled innocently enough "Mathematical Problems". In this talk, he outlined 23 problems facing mathematics that he believed should be researched during the coming century. His stature was such that the mathematical community went to work. The problems are as follows:

1. The Continuum Hypothesis
2. Determine the consistency of the axioms of arithmetic.
3. If two tetrahedra have equal bases and altitudes, are their volumes equal?
4. Is the shortest distance between two points a straight line?
5. The requirement of differentiability in Lie's continuous group of transformations
6. To give axiomatic treatment to mechanics and probability
- 7. Irrationality and transcendence of certain numbers**
8. Problems in Prime Number Theory
 - Riemann's Hypothesis, distribution of primes,
 - Goldbach's Conjecture, the number of twin primes
9. General Law of Reciprocity in Arbitrary Number Fields
10. Determination of the solvability of Diophantine equations with rational coefficients
11. Quadratic forms with algebraic coefficients
12. Extension of Kronecker's Theorem on abelian fields
13. Impossibility of the solution of the 7th degree general equation by functions of two variables
14. Finiteness of certain systems of functions
15. Rigorous foundation of Schubert's Enumerative Calculus
16. Relative positions of the branches of a plane algebraic curve
17. Positive definite forms expressible as quotients of sums of

- squares of forms
18. Polyhedra in 3-space
 19. Are the solutions to regular variational problems always analytic?
 20. The solvability of regular variational problems with suitable boundary conditions
 21. Existence of linear differential equations having a given monodromic group
 22. Uniformization of analytic relations by means for automorphic functions
 23. The calculus of variations

Hilbert's Seventh Problem dealt specifically with whether or not α^β is transcendental (or even irrational) given that α is a nonzero algebraic number and β is algebraic irrational number. In 1934 two mathematicians (Gelfond and Schneider) independently (and using different methods) proved the transcendence of α^β . The result has become known as the Gelfond-Schneider Theorem.

Aleksandr Gelfond (1906-1968), who was not even alive when Hilbert first posed this problem, was already well known at this time (despite his youth). In 1929, at the age of 23, he conjectured that if a_m and b_m are algebraic (for $m = 1, 2, \dots, n$) and if $\{\ln a_m\}$ are linearly independent (over the rationals), then $b_1 \ln a_1 + b_2 \ln a_2 + \dots + b_n \ln a_n \neq 0$. This was known as Gelfond's Conjecture.

In 1966, Alan Baker proved Gelfond's Conjecture, and it has since been named Baker's Theorem. Baker has taught at the prestigious University of Cambridge for the past 29 years.

Currently, research into transcendental numbers is focused on such "elementary" numbers as e^e , π^e , 2^e , π^π , and 2^π .

These numbers were unusual, but perhaps the strangest fact had not yet been revealed. In 1874, Georg Cantor proved that (1) the real numbers are uncountable, and (2) the algebraic numbers are countable. Therefore, the transcendental numbers, while appearing to be quite rare, are uncountably infinite. This was one of many results of Cantor that alienated him from the mainstream mathematical community. Kronecker in particular was

unimpressed with the crazy notion that the unit interval has as many points as the plane. Cantor himself was very surprised by this result. In a letter to Dedekind, he wrote,

Can a surface (say a square that includes the boundary) be uniquely referred to a line (say a straight line segment that includes the end points) so that for every point on the surface there is a corresponding point of the line and, conversely, for every point of the line there is a corresponding point of the surface? I think that answering this question would be no easy job, despite the fact that the answer seems so clearly to be "no" that proof appears almost unnecessary.

Three years later, upon his discovery that this was possible, Cantor wrote (again to Dedekind),

I see it, but I don't believe it!

He also had a falling out with his close friend Magnus Mittag-Leffler. It was Mittag-Leffler who was enemies with Nobel, the result of which is there is no Nobel Prize for mathematics. In 1885, Mittag-Leffler convinced Cantor to withdraw a paper he had submitted to *Acta Mathematica* because it was, in the words of Mittag-Leffler, "... about one hundred years too soon". Cantor's reply showed his dismay.

Had Mittag-Leffler had his way, I should have to wait until the year 1984, which to me seemed too great a demand! But of course I never want to know anything again about *Acta Mathematica*.

Cantor's continued troubles were either caused by (or caused) his many bouts with depression. He was institutionalized several times, and went through times of inactivity brought on by depressive episodes. One such time, he wrote to Mittag-Leffler,

... I don't know when I shall return to the continuation of my scientific work. At the moment I can do absolutely nothing with it, and limit myself to the most necessary duty of my lectures; how much happier I would be to be scientifically active, if only I had the necessary mental freshness.